The Exeter College Summer Programme
at Exeter College in the University of Oxford

Mathematics for Physical Sciences and Engineering

Course Description

This course will cover some important areas of calculus used in many physical sciences and engineering courses. It builds upon the calculus of functions of single variables, to cover multivariable calculus and functions of a complex variable. The former has applications in classical mechanics, thermodynamics and electrostatics, for example, while the latter is used extensively in advanced quantum theory and signal processing.

Prerequisites

It is essential that students taking this course have already studied basic single - variable calculus and basic linear algebra. The following topics will be assumed knowledge from the outset:

- o Differentiation (product, quotient, chain rules)
- o Integration (basic integration techniques, integration by parts and by substitution)
- o Vectors in cartesian coordinates, scalar (dot) and vector (cross) products

Some appreciation of the idea of a partial derivative would be helpful, but this will be covered in the first lecture for students who have not encountered any multivariable calculus before.

Teaching methodology

Lecture hours: 12 x 1.25 hours (15hrs)
Problem classes: 6 x 1.25 hours (7.5hrs)
Tutorials: 4 x 1.25 hours (5hrs)

The problem classes cover a set of problem sheets based on the lectures, to be completed in advance of each class by the students.

Performance evaluation

Final examination: 60%
Assessed problems: 30%
Participation and attendance: 10%

Core reading

The recommended text for lectures 1-8 is: H. M. Schey, d*iv, grad curl and all that*, 2005, Norton
The presentation of the material will follow Schey's book fairly closely, **and will be at a similar level of difficulty and mathematical rigour.**

This material, plus that of lectures 9-12, is covered also in most 'mathematics for science' textbooks, including:

- K.F. Riley and M.P. Hobson, *Essential Mathematics for the Physical Sciences*, 2011, Cambridge.
- M. L. Boas, *Mathematical Methods in the Physical Sciences*, 2005, Wiley.
- G. B. Arfken and H. J. Weber, *Mathematical Methods for Physicists,2012,* Academic Press.

Lecture List

1. Foundations
    a. Derivatives and partial derivatives
    b. Integration as a Riemann sum
    c. Review of basic vector algebra
2. Multiple integrals
    a. Definition
    b. Calculation via nested integrals
    c. Change of order of integration
3. Change of Variables Theorem
    a. Justification
    b. Polar and spherical polar coordinates
    c. Applications
4. Line integrals
    a. Motivation via concept of work
    b. Calculation by parametrisation
    c. Green's Theorem in the plane and path independence
5. Surface integrals
    a. Motivation via concept of flux
    b. Calculation by parametrisation
6. Divergence
    a. Definition in terms of infinitesimal flux
    b. Calculation via partial derivatives in Cartesian coordinates
    c. Divergence Theorem
7. Curl
    a. Definition in terms of infinitesimal circulation
    b. Calculation via partial derivatives in Cartesian coordinates
    c. Stokes's Theorem and path independence of line integrals
8. Gradient
    a. Motivation from path independence of line integral
    b. Potentials
9. Functions of a complex variable
    a. Differentiability
    b. Cauchy-Riemann equations
10. Contour integrals
    a. Line integrals in the complex plane
    b. Cauchy's integral formula
11. Taylor and Laurent series
    a. Expansion of analytic functions as Taylor series
    b. Expansion as Laurent series around a pole
    c. Residues
12. The Residue Theorem
    a. Application to integration