Exeter College Oxford Summer Programme
An Introduction to Fluid Dynamics

Course Description

This course introduces students to the mathematical theory of fluids via the Navier-Stokes Equations. The equations can be used to successfully model almost any fluid on Earth, but our mathematical understanding of them remains limited. So much so, that a $1-million prize exists for anyone that can help to further our understanding of problems involving vortex reconnection, turbulence and whether or not the equations are 'well-posed'. We will look at examples in inviscid flow theory which provide insight into physical phenomena such as flight, vortex motion, and water waves. We will also explore the basic fluid dynamics necessary to build mathematical models of the environment in which we live, focusing on problems such as climate change, pollution, or the spread of infectious aerosol droplets within our buildings.

Synopsis

Navier-Stokes Equations:
Incompressible flow. Convective derivative, streamlines and particle paths. Euler's identity and Reynolds' transport theorem. The continuity equation and incompressibility condition. Cauchy's stress theorem and properties of the stress tensor. Cauchy's momentum equation. The incompressible Navier-Stokes equations.

Inviscid Flow:
Euler's equations of motion for an inviscid fluid. Irrotational incompressible flow; velocity potential. Two-dimensional flow, stream function and complex potential.

Vortex Motion:
The vorticity equation and vortex motion. Line sources and vortices. Method of images.

Water Waves:
Water waves, including effects of finite depth and surface tension. Dispersion, simple introduction to group velocity.

Prerequisites

A degree of mathematical background will be required. In particular, experience of calculus is a necessity, and previous exposure to mathematical modelling via partial differential equations would be helpful. The course is best suited to anyone studying a mathematical discipline such as maths, physics or engineering, that wishes to apply the tools they have learned to solve real-world problems.

Teaching Methods and Assessment

12 x Lectures

- Vector Calculus Essentials x2
- Navier-Stokes Derivation x2
- Millennium Problems x1
- Inviscid Flow Theory x2
- Vortex Motion x1
- Water Waves x2
- Ocean Pollution x1
- Climate Models x1

6 x Seminar Classes

- Vector Calculus Examples Class
- Fluid Dynamics Experiments
- Millennium Problems Presentations (15% of final grade)
- Inviscid Flow Theory Examples Class
- Water Waves Examples Class
- Environmental Issues Presentation (15% of final grade)

4 x Tutorials

- Vector Calculus Problem Set (10% of final grade)
- Inviscid Flow Problem Set (10% of final grade)
- Vortex Motion Problem Set (10% of final grade)
- Water Waves Problem Set (10% of final grade)

Final Assessment: 3-hour written examination (30% of grade)