Exeter College Summer Programme
Mathematical Modelling - Dr Tom Crawford

Course Description

This course will introduce the key principles of mathematical modelling, which will then be explored through several real-world examples in disease modelling, environmental planning, and population dynamics. The techniques of calculus are essential, although core concepts such as differential equations will be revisited. We will also touch on the mathematical models used in data science, particularly the techniques of principal component analysis and clustering, both essential tools in machine learning models.

Course Objectives

The main objective is to provide real-world applications and examples of mathematical models. Once the key ideas of mathematical modelling have been understood, the technique can be applied to almost any situation to gain insight into how a system behaves. The course will provide a wide variety of examples as a means to showcase the versatility of the tools, with the opportunity for students to develop their own models to be applied to subjects of their choosing. By the end of the course, students will have a new-found appreciation of the power of mathematical modelling, and be equipped with the skills needed to use them in their own work in the future.

Teaching Methods and Assessment

12 x	1.25hr Lectures	(15hrs)
6 x	1.25hr Seminars	(7.5hrs)
4 x	1.25hr Tutorials	(5hrs)

The lectures aim to present the mathematical techniques, while the seminars are an opportunity to discuss the models and their outcomes in more depth. Each seminar will be based around a particular model, with further reading from the academic literature. For each group of 3 lectures, students will complete a problem set with questions based on the mathematical techniques presented in lectures. The solutions to these problems will be discussed in the tutorials (one-on-two meetings between a tutor and two students).

Prerequisites

Students will need some background in calculus and linear algebra - for example an introductory course in each at university level.

Lecture Topics

Group I (Differential Equations)
1 Calculus Primer
2 Differential Equations I
3 Differential Equations II

Group II (PDEs)
4 Partial Differentiation
5 Partial Differential Equations I
6 Partial Differential Equations II

Group III (Linear Algebra)
7 Linear Algebra Primer
8 Eigenvalues and Eigenvectors
9 Spectral Theorem

Group IV (Data Analysis)
10 Phase-Plane Analysis
11 Principal Component Analysis
12 Clustering

Required Readings
All readings will be made available online before each seminar.

1. Stability / Chaos Theory
2. SIR Model
3. Heat Equation / Wave Equation
4. Investments / Stocks
5. Animal Population Dynamics
6. Ocean Pollution

Assessment:
Project work developing your own mathematical model (50%)
A final 3hr written examination (40%)
Participation in seminar and tutorial discussions (10%)

Project (50%)
Using the concepts developed in the lectures, and the models discussed in the seminars, you will derive your own mathematical model for a topic of your choice. Previous examples include: projectile motion of a badminton shuttlecock, a ranking algorithm for Valorant players, calculating the carbon footprint of air travel, modelling waves for ideal surfing conditions. There will be opportunities to discuss your ideas and progress during the seminars.

The Exam (40%)
The final exam will consist of eight questions, from which you must answer six. These will be based on the problem sets and will be grouped with 2 questions from each lecture grouping: Differential Equations, PDEs, Linear Algebra, Data Analysis.

To be fully prepared for this exam, you must complete the problem sets for each tutorial, and revise the material covered in lectures. The exam will test your knowledge and understanding of the material covered in lectures and tutorials.

Participation (10%)
Discussion is a crucial part of this course, so please come to each seminar and tutorial with questions and comments of your own. Also, remember that the quality of your contributions is more important than how often you contribute. Finally, regular attendance is expected and required.